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## SYSTEM FOR GEOMETRICAL MODELING

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The present invention relates in general to computational modeling of geometric structures  
9 involving the disclosures submitted with prior copending application Serial No. 09/109,725 filed  
10 July 2, 1998, with respect to which the present application is a continuation-in-part.

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### BACKGROUND OF THE INVENTION

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The representation of physical bodies using techniques associated with computational  
15 geometry or geometrical modeling, is already known in regard to design, visualization and  
16 manufacture of structural products or objects involving various limitations such as surface  
17 properties, shapes and part relationships. A lot of attention has been given to "Smart Product  
18 Models", "Simulation Based Design", "Integrated Design Environment", and "Integrated Product  
19 Data Environment". It has become evident that in "integrated environments" geometry plays a  
20 vital role as the common element in the design and analysis of complex systems. In essence,  
21 computational geometry is the closest data representation of the actual product of any piece of  
22 information. The form of geometric representation can determine the level of effort involved in  
23 generation of analysis data, the means to communicate the design for product review and  
24 mockup, and the ability to present the product for manufacturing. In essence, the "smart" product  
25 model more closely represents real products in function, form, performance, and physical  
26 characteristics.

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In regard to the present invention, a technical publication entitled "Framework For  
29 Modeling Complex Structures" was published after the filing of the aforementioned copending  
30 parent application. Some of the content of such publication was extracted and transferred to the

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1 specification and drawings of the present application. The remainder of such publication is  
2 incorporated herein by reference, including terminology definitions.  
3

4 SUMMARY OF THE INVENTION

5 In accordance with the present invention, geometric and polynomial elements are utilized  
6 to represent physical structures, modeled as topological views of analytic functions, whose  
7 bounding elements are related to connectors, defined in terms of mathematical algorithms to  
8 bounding elements of other topological views, whereby geometrical modeling, discretization for  
9 computational analysis, and multi-discipline integration of such structures may be performed by  
10 computer aided design involving computational techniques. The topological view of a structural  
11 model is formed with one outer element boundary together with one or more inner element  
12 boundaries for which unique physical and/or solution properties or attributes may exist. Inner  
13 boundaries which overlap or intersect with each other or with outer boundaries are excluded.  
14 Types of topological views include, but are not limited to, bounded surface regions with  
15 parametric edge element boundaries and bounded volume regions with bounded surface region  
16 element boundaries. Spatial relationships between boundary object elements, called connectors,  
17 are defined as algorithms of multi-dimensional functions or multidimensional mappings.  
18 Connections and topological views combine to create robust and spatially connected logical and  
19 functional structures along with their associative properties.  
20

21 The present invention involves the role of related software developments in geometric  
22 modeling, discretization for computational analysis, and multi-discipline integration. Such  
23 developments provide a mathematical framework for geometry, grid data structures, analysis  
24 characterization, and behavior modeling. A new geometry topology framework provides a  
25 unique modeling capability to represent typical related geometry as well as provide more  
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1 complete functionality for discretization, evaluation, and modeling of complex boundary  
2 representations. Also provided is the capability to facilitate the work of an integrated product  
3 team. A unique role of the foregoing software developments is performed in the design and  
4 analysis of complex structures.  
5

6 BRIEF DESCRIPTION OF DRAWING

7 A more complete appreciation of the invention and many of its attendant advantages will  
8 be readily appreciated as the same becomes better understood by reference to the following  
9 detailed description when considered in connection with the accompanying drawing wherein:

- 10 FIG. 1 is a composition functional diagram;  
11  
12 FIG. 2 is a surface and grid mapping diagram;  
13  
14 FIG. 3 is a surface and solution data mapping diagram;  
15  
16 FIG. 4 is a representation of multiple topology geometry;  
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18 FIG. 5 is a house arrangement diagram;  
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20 FIG. 6 is a GOBS Unified Modeling Language diagram of the Structure class;  
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22 FIG. 7 is a GOBS Unified Modeling Language diagram of the Spline class;  
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24 FIG. 8 is a GOBS Unified Modeling Language diagram of the Surfaceclass;  
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26 FIG. 9 is a GOBS Unified Modeling Language diagram of the Face class;  
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28 FIG. 10 is a GOBS Unified Modeling Language diagram of the Solid class;  
29  
30 FIG. 11 is a GOBS Unified Modeling Language diagram of the Topological View class;  
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32 FIG. 12 is a GOBS Unified Modeling Language diagram of the Property class  
33  
34 FIG. 13 illustrates boundary element candidates for construction of a topological view of  
35 surface type of physical structure from which data is extracted for geometrical modeling, and  
36 physical properties are associated in accordance with the present invention;

1 FIG. 14 illustrates a connector object of type CoEdge utilized for connecting topological  
2 views of type face;  
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4 FIGS. 15A and 15B are two dimensional domain views of parametric space illustrating the  
5 CoEdge relationship between different topological views shown in FIG. 14.  
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7 FIG. 16 illustrates a connector of type surface connecting overlapping topological views  
8 of type surface;  
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10 FIG. 17 illustrates two connected topological views of type volume where a topological  
11 view of type face bounds both.  
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13 FIG. 18 is a block diagram of robust geometry showing application of topology elements  
14 and connectors necessary for "smart" product modeling.  
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#### DETAILED DESCRIPTION OF PREFERRED EMBODIMENTS

16 The present invention involves product modeling through use of representational methods  
17 based on the assumption that all disciplines need product model data; that a common required  
18 representation exists; and that the product model contains sufficient information to convey design  
19 intent and detail for all disciplines. A fundamental premise associated with such methods is that  
20 product model data is composed of geometry, performance and modeling process, applicable to  
21 design and engineering analysis, and that such a product model can be composed.  
22

23 Characterization of shape, behavior and content are fundamental to the product modeling  
24 process. Correct shape representation is dependant on the ability to define connectedness of the  
25 structure through computational methods and not through visual reference. Pursuant to the  
26 present invention, use of mathematical methods and the relationships therebetween provides a  
27 framework for "smart" product model development necessary for multi-discipline integration in a  
28 fully capable design environment. The foregoing referred to framework consists of: (1) a

1 Geometry and Engineering Mathematical Library (GEML) as a mathematical foundation for  
2 modeling; (2) Geometry Object Structure (GOBS) utilizing such mathematical foundation to build  
3 unique topologically connected models and allow for development of robust geometry necessary  
4 for smart product modeling; and (3) Leading Edge Advanced prototyping For Ships (LEAPS)  
5 which involves a combination of GOBS modeling with property and performance data. The  
6 application of the aforesaid framework is the basis from which the environment for designing and  
7 assessing complex systems is created.

8  
9 In regard to (1) GEML, which provides a mathematical basis for modeling, the object of  
10 such library is to: (a) develop, import, export, modify and evaluate spline based mathematics; (b)  
11 develop, modify, adapt, grid or mesh data with associated spline functions; and (c) store resultant  
12 geometry, grid, and analysis data in a common framework with dependencies maintained. Using  
13 these objects, a GEML based framework contains and controls all information relating to  
14 geometry, grids, and analysis. Since analysis is dependent on grids, and grids on geometry, the  
15 coupling of these elements in an object oriented data structure is accommodated. GEML provides  
16 for flexible representation and object dependency. It includes such features as geometry varying  
17 with time; mapped geometry; grids defined in the geometry domain, and n-dimensional functions  
18 for analysis characterization and behavior modeling.  
19

20  
21 In order to model geometry, grids, and analysis data it is necessary to have a mathematical  
22 basis with facilitates representation of all three and allow for both implicit and explicit coupling  
23 of the data. A fundamental element in GEML is the notion of n-dimensional spline functions,  
24 combined with composition, select, and affine functions. N-dimensional splines in GEML allows  
25 for the modeling and interpolation of any number of geometric and analysis variables; and by any  
26 number of independent parameters such as time, frequency, or iteration. Such N-dimensional  
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splines are the building block for dynamic, variational geometry and representation of analysis data as a function. N-dimensional analysis functions are also ideally suited for use as performance behavior models. Consider a traditional spline surface and representation thereof extended by GEML to N-dimensions, as respectively depicted in the followings equations (1) and (2):

$$(x, v, z) = f(u, v)$$

### **Equation 1 - Traditional CAD Surface**

## vs. GEML

$$(x, y, z, Vx, Vy, Vz, Cp, \text{temp}, \dots) = g(u, v, \text{time}, \dots)$$

### **Equation 2 - GEML N-Dimensional Function**

Of the GEML functions, the spline and composition functions provide the essential building blocks for geometry and analysis characterization and coupling. Additionally, the ability to map such functions to each other extends affords flexibility by removing parametric issues as a modeling constraint.

Consider an example wherein a surface is mapped to another surface. In this example, the mapping of one geometry to another is described by the composition of two functions. The first, parent geometry, is represented as a tensor product spline. The second, a mapped function, is also a tensor product spline entity, but its range is in the domain of the parent geometry. Composition functions are not limited by the dimensionality of the parent or base function. In the case of a typical CAD surface,  $(x,y,z)=f(u,v)$ , the parent representation would not allow for the inclusion of the notion of a surface changing with time. GEML, because of its non-dimensional nature, adds the ability to represent the parent geometry as a surface changing with time, i.e.  $(x,y,z)=f(u,v,\text{time})$ . Thus, the domain of the parent geometry is an  $n$ -dimensional cube  $R$ . The domain of the mapped surface function is another  $n$ -dimensional cube  $D$ . In such case, the

mapped surface will not change in its location on the parent surface but its 3D image will change as the parent changes.

The foregoing reference to this geometry model is depicted in FIG. 1 where the mapping  $f: R \rightarrow E^j$  is the parent geometry of the particular surface component and  $g: D \rightarrow R$  is a surface mapping defining the relevant piece of the parent geometry. The notion of a composition of function arises from the mapping of one function with another such that the evaluation of that combined function behaves in a fashion similar to a chain rule.

The foregoing concept may be extended by forming a composition function which maps analysis data to geometry. Such mapping giving the ability to interrogate the relationship between performance and geometry because they are directly related and dependent.

A single n-dim function could contain;  $(x, y, z, Vx, Vy, Vz, Cp, \text{temp}) = g(u, v, \text{time})$   
 Now suppose we separate the geometry out and  $(x, y, z) = f(u, v)$   
 create a second function containing the analysis data.  $(u, v, Vx, Vy, Vz, Cp, \text{temp}) = g(s, t)$   
 By composing functions we perform the following;

$f: (u, v) \rightarrow (x, y, z)$  and  $g: (u, v, r1, r2, r3) \rightarrow (s, t)$   
 $f \circ g(s, t) \rightarrow (x, y, z, r1, r2, r3)$

The notation  $f \circ g(x)$  means "function  $f$  composed with function  $g$ ," and could be written as  $f(g(x))$ .

The foregoing example demonstrates that n-dimensional splines and composition functions allow for the use of mapping from space to space. A surface can lie on another surface, a volume is contained within a volume, or a function can define the relationship between two discrete curves, surfaces, or volumes. A mapped surface may be gridded independent of the parent geometry to change with any parameter if necessary as shown in FIG. 2. By including the n-dimensional mapped functions, grid can be evaluated and the solution set returned and interpolated as a spline mapped to the geometry as in FIG. 3.

In regard to (2) GOBS, a new model for the construction, representation, and interrogation of complex product models is provided. The GOBS model purports that product model data is defined and represented as a ‘view’ of implicit and explicit geometric and analytic entities. Views are compositions of entities whose relationships and attribute associations define the entity. This is contrary to most representations where the geometry defines the view and the object simultaneously. FIG. 4 depicts multiple geometry representations typical in product modeling as performed using computer aided design (CAD). The geometry depicted illustrates the many forms used in CAD systems today; which contain the same mathematical basis, but the use of which topology of the model can vary widely. Each topology has benefits and constraints in product modeling but none correctly represents real physical structure. For example, a typical CAD b-rep solids representation treats the ship depicted in FIG 4 as a collection of solid blocks each containing its own entities without knowledge of its relationship to the other entities. The more common product model representation using trimmed surfaces generally provides the most visually correct representation but loses all associations between surfaces. For example, there is no knowledge about the relationship between hull and deck and that the two adjacent surfaces are connected. The topology model based on GOBS provides the means to resolve such issues. Geometric entities can be explicitly connected by their edges and discretization can occur across faces while simultaneously resolving parameterization issues across boundaries. Using these connections in combination with topological views, both solid and surface views can exist simultaneously while adding knowledge of product structure relationships.

Another example of GOBS topology is illustrated in FIG. 5 with a universally understood example. Here we have the common notion of a house arrangement formed by a kitchen, family room, bedroom, and hallway. Semantics, features, and shape recognition typically resolve any

confusion about which region belongs with which space but geometrically and analytically they are undefined. The product structure must be resolved visually. To ask the question, "What is the area of the kitchen floor?", would generally suggest to the CAD operator that the kitchen floor must be modeled with different geometry than that used to model the family room and hallway. Thus, the area of the kitchen floor becomes the area of the surface. This assumption of geometry and space falls apart when other relevant disciplines become involved. During construction of a house, the plywood floor panels, carpeting, and drywall, are independent of these other spatial designations. So to layout a grid of plywood panels on the floor surface would require a different geometric representation. The issue becomes more complicated with more complicated models.

Ask the question, "What is the area of the roof of a car"? To resolve these issues, it is important to realize that semantics can be universally understood but not explicitly defined. Thus, everyone recognizes that the roof of a car is the somewhat flat region above the space where you sit, but where the roof ends and the roof pillars begin is not defined. Current topology models do not allow for the construction of a product model that can define the kitchen floor or the roof of a car without unique, explicit, and redundant representation.

The GOBS structure allows for the extension of all representations into a common topology using non-manifold models with support for manifold and non-manifold "views". Such topology will also allow for future extensions, which will include additional n-dimensional objects and composition functions. These extensions will allow more flexibility in deserialization for physics based analysis, variational or time dependent analysis mappings, and allow more complete representations of product structure in general. The GOBS model utilizes a unique structure that allows geometry to be treated as a view of analytic functions, which satisfy the constraints of typical geometric entities. An example is the representation of trimmed surfaces.

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1 Currently CAD systems today consider a trimmed surface to be composed of a **surface geometry**  
2 bounded by a single outer boundary and any number of inner boundaries. This model does not  
3 allow the surface to be used as an entity to any other trimmed surface. It requires that a copy of  
4 that surface be made. GOBS, on the other hand, allows *topological views* of a trimmed surface  
5 where the view contains reference to one surface, one outer loop, and any number of inner loops.  
6 Thus, trimmed surface topological views of the kitchen floor, family room, hallway, etc. could be  
7 constructed using a single surface covering the entire floor and the boundaries depicting space, or  
8 any other subdivision of the floor surface. If these spaces do not satisfy the constraints of  
9 *topological views* they can be grouped as *common views*. Examples of additional spatial  
10 decompositions in *common views* could be “livable space”, “carpeted space”, “arrangeable  
11 space”, etc.

14 GOBS includes relational elements that allow for the connection of entities through  
15 geometric reference and related through algorithmic functions. The wall between the family  
16 room and the bedroom as depicted in FIG. 5 is attached to the floor. The wall may span multiple  
17 floors but it contains a geometric intersection with the floor. To better understand how these  
18 relational elements are composed, the following GOBS definitions of the class elements and their  
19 associations is presented.

21 **Factory (GobsFactory)**

22       Manages Gobs *Structures*

23 **Structure (GobsStructure)**

24       A *Structure* object creates and manages all Gobs objects that geometrically  
25 describe the design object. A *Structure* can be viewed as a collection of connected  
objects or entities that compose a part or structure.

25 **Common View (GobsCommonView)**

26       A *Common View* object groups *Topological View* objects and/or other *Common*  
27 *View* objects into a logical view of a *Structure* object. Objects may be members of  
multiple *Common Views*.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28

## 1 Topological View (GobsTopologicalView)

2 A *Topological View* object is a *Surface* object, a *Face* object, or a *Solid* object.  
3 *Topological Views* have *Characteristic* and *Property* objects.

## 4 Solid (GobsSolid)

5 A *Solid* object is a boundary-representation solid whose boundaries are *Oriented*  
6 *Closed Shell* objects.

## 7 Face (GobsFace)

8 A *Face* object is a part of a *Surface* object defined by *Edge Loop* objects.

## 9 Surface (GobsSurface)

10 A *Surface* object is a non-uniform rational b-spline representation of a surface.

## 11 Pcurve (GobsPcurve)

12 A *Pcurve* object is a parametric curve on a *Surface* object. Its range is the domain  
13 of the *Surface* it is mapped to.

## 14 Ppoint (GobsPpoint)

15 A *Ppoint* object is a point on a *Pcurve* object.

## 16 CoPoint (GobsCoPoint)

17 A *CoPoint* object is a collection of *Ppoint* objects that are the same point in model  
18 space.

## 19 Edge (GobsEdge)

20 A *Edge* object is a segment of a *Pcurve* object defined by two *Ppoints*.

## 21 CoEdge (GobsCoEdge)

22 A *CoEdge* object is collection of *Edge* objects that represent the same model space  
23 edge. A *CoEdge* contains a single n-dimensional function that contains the  
24 domain space of each member *Edge* as range variables of its function.

## 25 Edge Loop (GobsEdgeLoop)

26 A *Edge Loop* object contains a series of connected *Edge* objects that form a closed  
27 loop.

## 28 Oriented Closed Shell (GobsOrientedClosedShell)

29 A *Oriented Closed Shell* object is a collection of *Face* objects that form a closed  
30 shell.

## 31 Cartesian Point (GobsCartesianPoint)

32 A *Cartesian Point* is a point in Cartesian space.

## 33 Characteristic (GobsCharacteristic)

34 A *Characteristic* object contains *Property* objects and/or other *Characteristic*  
35 objects.

## 36 Property (GobsProperty)

37 A *Property* object contains an attribute's name and data.

1 The foregoing classes and their associations are diagrammed in FIGS. 6-12, using Unified  
2 Modeling Language notation.  
3

4 The present invention involves the use of connectors (i.e. GOBS CoEdges) and  
5 topological views (i.e. GOBS Topological Views).  
6

7 FIG. 13 illustrates by way of example a topological view of a two-dimensional base  
8 surface 10 of which four parametric boundary elements exist. Several closed loop boundaries 12,  
9 14, 16, and 18 are defined as composition functions on surface 10. A total of eight possible  
10 topological views could be composed from these topology elements. Four trimmed surfaces with  
11 no inner boundaries and outer boundaries 12, 14, 16, and 18 are possible. Four trimmed surfaces  
12 with a single outer boundary 12 and their respective inner boundaries include (14 and 18), (14),  
13 (16), and (18). Since each pair of inner loops (14 and 16) or (16 and 18) are intersecting as  
14 shown in FIG. 13, such pairs of inner boundaries are not used for mapping pursuant to the present  
15 invention, which excludes self intersecting boundaries as well as any boundary intersecting  
16 another boundary.  
17

18 Fundamental to the GOBS topology, the CoEdge provides a unique role in the connection  
19 of boundaries necessary for robust topologies. A CoEdge's function is to know all edges located  
20 on each surface and declares them to be equivalent in Cartesian space. It also contains an n-  
21 dimensional spline function which maps the parameterization of each edge into a single function.  
22 This allows for the continuity of points along one surface to migrate to another surface without  
23 having to perform closest point approximations.  
24

25 Pursuant to this invention, a connector object is illustrated in FIG. 14 showing two surface  
26 objects 20 and 22 connected to each other along a single intersection 24 between the curved  
27 surfaces of such objects, referred to as a CoEdge connector, which is expressed in terms of a  
28

1 multi-dimensional function that maps both the physical intersection and the parametric domains  
2 of their respective edge boundaries.

3 Domain plots of the surfaces 20 and 22 and the underlying topology elements of the CoEdge  
4 connector are respectively diagrammed in FIGS. 15A and 15B with bounded parametric edge  
5 curves 26 and 28 respectively having parametric point boundaries 30, 32, and 34, 36. These  
6 parametric points, called GOBS Ppoints, are defined as a parametric location **n** in the domain of  
7 curves 26 and 28. A connector object of type GOBS CoPoint maps 30 and 34 to a Cartesian  
8 location in space and another CoPoint similarly maps 32 and 36 to a location **n** in Cartesian space.  
9 The CoPoint declares the two parametric Ppoints to be the same location in Cartesian space much  
10 the same as the CoEdge does to parametric edges on different surfaces.

12 Connector objects are not limited to edge boundaries of topological view representation of  
13 type surface or face as that illustrated by way of example in FIGS. 13, 14, 15A and 15B. In regard  
14 to the connector function associated with the topological view diagrammed in FIG. 16, the  
15 connector function ( $g$ ) is:  $(i,j) \rightarrow (u, v, s,t)$ . Here a connector object of type surface 42 is  
16 connecting and mapping the space between two overlapping surfaces 38 and 40. Three-  
17 dimensional solids or volumes, 44 and 46, illustrated in FIG. 17 are likewise related by a  
18 connector of type surface 46. The connector objects in FIG 17 would have a functional  
19 relationship appearing, given the volume 44 defined as the spline function  $(x_1, y_1, z_1) = g(i,k,j)$   
20 and the volume 46 defined as the spline function  $(x_2, y_2, z_2) = f(u,v,w)$ , as  $(i,j,k,u,v,w)=r(l,m)$ .  
21 The construction of the connector would ensure that for a given  $(l,m)$  on function  $r$  a set of  
22 parametric locations  $(i,j,k,u,v,w)$  would be returned such that if evaluated against their respective  
23 functions  $g$  and  $f$  would return the same Cartesian location  $(x,y,z)$ .

DRAFT PCT 160

1 Pursuant to the present invention, topological views of different geometrical structures  
2 respectively illustrated by way of example in FIGS. 14, 16 and 17, include connector objects  
3 providing logical and explicit connections for mapping domain to domain and domain to range  
4 between structural objects without being restricted by current geometric modeling methods to the  
5 dimensional function ( $R^3$ ) denoting bounded regions of three dimensional Cartesian spaces.  
6 Based on the foregoing described topological view representations and the described mapping  
7 procedure associated therewith, a framework is established for different structures from a design  
8 environment for computational and geometrical modeling purposes.

9  
10 FIG. 18 diagrammatically summarizes the procedure of the present invention as  
11 hereinbefore described, applicable to computer aided, geometric product modeling denoted by  
12 reference numeral 48. Such procedure involves product model representation; composed of  
13 connectors 58 and topological views 50. Such topological views are defined by outer boundary 52  
14 and zero or more non-overlapping inner boundaries 54, whereby such boundaries are composed  
15 of individual boundary elements 56, so as to associate unique properties 62 within regions of the  
16 base geometry 60. The composition of product structure is made robust and associative through  
17 connectors 58 which mathematically relate boundary elements 56 across base geometry 60  
18 domains.  
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20  
21 Obviously, other modifications and variations of the present invention may be possible in  
22 light of the foregoing teachings. It is therefore to be understood that within the scope of the  
23 appended claims the invention may be practiced otherwise than as specifically described.

24 What is claimed is:  
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